

Theorem 1 [Bor].

Suppose P is a Coxeter polytope with Coxeter diagram Σ , and p is the face corresponding to an elliptic subdiagram σ of Σ that has no A_n and D_5 component. Then p is itself a Coxeter polytope.

A node of Σ attaches to σ if it is joined to some node of σ by an edge of any type.

Let p be a face of P . Let a and b be facets of p such that $a = A \cap p$, $b = B \cap p$, where A and B are facets of P . Denote by $\angle ab$ the dihedral angle of p formed by a and b .

Theorem 2 [All]. Under the hypotheses of Theorem 1:

- (1) If neither A nor B attaches to σ , then $\angle ab = \angle AB$.
- (2) If just one of A and B attaches to σ , say to the component σ_0 , then
 - (a) if $A \perp B$ then $a \perp b$;
 - (b) if A and B are joined by a single edge and adjoining A and B to σ_0 yields a diagram B_k (resp D_k , E_8 or H_4), then $\angle ab = \pi/4$ (resp $\pi/4$, $\pi/6$ or $\pi/10$);
 - (c) otherwise, a and b do not meet.
- (3) If A and B attach to different components of σ , then
 - (a) if $A \perp B$ then $a \perp b$;
 - (b) otherwise, a and b do not meet.
- (4) If A and B attach to the same component of σ , say σ_0 , then
 - (a) if A and B are not joined, and $\sigma_0 \cup \{A, B\}$ is a diagram E_6 (resp. E_8 or F_4), then $\angle ab = \pi/3$ (resp. $\pi/4$ or $\pi/4$);
 - (b) otherwise, a and b do not meet.

[All] D. Allcock, *Infinitely many hyperbolic Coxeter groups through dimension 19*, *Geom. Topol.* 10 (2006), 737–758.

[Bor] R. Borcherds, *Coxeter groups, Lorentzian lattices, and K3 surfaces*, *Int. Math. Res. Notices* (1998), 1011–1031.